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A PARAMETRIC STUDY OF THE THERMIONIC DIODE SYSTEM
FOR LARGE NUCLEAR-ELECTRIC POWERPLANTS

IN SPACE VEHICLES

By Daniel T. Bernatowicz

Lewis Research Center Cleveland, Ohio

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SUMMARY

A parametric analysis of thermionic diode systems for large space powerplants is presented, and the diode systems are compared with the Rankine turbogenerator system on the basis of radiator area. Diode performance is computed with the height of the potential barrier above the Fermi levels in the cathode and anode as parameters. Heat radiation, heat conduction, electron heating, and joulean heating are taken into account. Plasma losses are neglected. The minimum possible radiator area is determined for several configurations as a function of cathode and radiator temperatures.

Gas is not considered as the heat-transfer medium for heating or cooling thermionic diodes for large space powerplants because many high-temperature ceramic-to-metal seals are needed to contain the gas. Also, the low volumetric heat capacity of a gas means only a fraction of the diodes will operate with the best temperatures.

Use of a liquid metal as the heat-transfer medium means temperatures in the diode system may be limited by corrosion problems just as in the Rankine cycle turbogenerator. In this study, it is assumed that liquid-metal temperature is limited to the same value in both systems. If the liquid metal is used to heat the cathodes, the diode system will be restricted to peak temperatures the same as those for a turbogenerator, and the diode system will require a larger radiator than a turbogenerator with a conversion efficiency of 0.7.

Incorporating the diodes into a reactor fuel element and using a liquid metal to cool the diodes allows the diodes to operate at a higher peak temperature for the same corrosion limit. However, using radiative heat transfer from the anode to the liquid metal to eliminate short circuiting penalizes the diode system sufficiently that the same radiator area is required as in the turbogenerator, and further, diode power density is so low that a very large reactor and shield are necessary. Conductive heat transfer through an electrical insulator to the coolant permits substantial savings in radiator area, up to 50 percent at a cathode temperature of 1700° K and up to 80 percent at 2300° K.

INTRODUCTION

This study of the thermionic conversion system for use on space vehicles is an analytical survey of the possible designs to outline the regions of highest potential performance and to indicate by means of a simplified comparison with the Rankine cycle turbogenerator whether either offers clear advantage over the other. For power systems that will be used in space the mass of the system is the ultimate criterion for optimization or comparison, but it is a difficult parameter to eval-In a survey such as this it is expeditious to look at a parameter that is easier to evaluate yet strongly influences the mass. In nuclearelectric powerplants for use in space the heaviest single component dependent on powerplant performance is the radiator required to reject the waste heat. For powerplants capable of supplying electric power for propulsion of large space vehicles, upwards of 1 megawatt, the mass of the radiator is estimated to constitute one-third to one-half of the total mass. The mass of the nuclear reactor shield is estimated to be of roughly the same magnitude, but because its size depends primarily on the size of the reactor, it is relatively fixed in weight. The shield mass will not differ much among various conversion schemes except in those that require a large reactor.

Reference 1 shows that the thermionic converter is theoretically capable of high efficiency. However, for nuclear-electric systems for use in space, efficiency alone is not a suitable index of performance. The radiator size depends not only on the efficiency of the cycle but also on the temperature of the radiator. The high premium on high radiator temperature forces the cycle toward operation at low efficiency. Therefore the high efficiencies that might be realized in ground applications, where the anode temperature could be as low as ambient, are not germane to the evaluation of the thermionic diode for use in space.

In this analysis made at the Lewis Research Center radiator area per unit of power output for a nuclear thermionic system is computed for a range of system temperatures and diode performance parameters to outline the range of these variables that yields minimum radiator area. Radiator area requirements for the thermionic and the Rankine cycle turbogenerator systems are compared. Whether the reactor core size will be limited by diode performance is considered. The range of temperatures studied is 1700° to 2300° K for the cathode and 1100° to 1800° K for the anode.

MECHANICAL CONSIDERATIONS

Some examination of the mechanical aspects of the design is in order so that some selectivity may be exercised and the effort narrowed to those design concepts that appear more practical at this time. Also, discussion of the mechanical aspects is required to justify the manner in which the diode systems and the turbogenerator systems are to be compared.

Location of Diodes

Thermionic diodes can be incorporated into large space power systems in three general ways. One is mounting them in the radiator so that the anode, perhaps augmented with fins, would provide the radiating area. The cathodes would be heated by a heat-transfer loop from the reactor. Another scheme is placement of the diodes in the reactor with the fuel elements as the cathodes, whereby the heat would be used where it is generated. The anodes would be cooled by a loop to the radiator. In a third method the diodes are separate from both the reactor and the radiator and two heat-transfer loops are used.

In a special case that can be classified as a combination of the first and second the diodes are mounted on the outer surface of the reactor so that the cathodes can be heated by conduction from the reactor and the anodes cooled by radiation to space. Such a system has the advantage that it is completely static, but the size of the reactor is determined by the surface area required for heat rejection, and for megawatts of power the reactor size is prohibitive. Therefore this system is not considered in this analysis.

Direct Heating or Cooling of Electrodes

The implications of requiring that heat be transferred to or from the electrodes by a heat-transfer fluid flowing in direct contact with the electrodes is considered in this section. Electrical short circuiting eliminates liquid metals, and probably all liquids for the temperature range of interest. Short circuiting is not a problem with gas as the heat-transfer fluid, but the low heat content per unit volume and poor heat transfer coefficient of gases will result in (1) large temperature variations, which will compromise the performance of the diode, and (2) large heat-transfer area, which will make necessary large and heavy components throughout the system. Furthermore, the development of a gas-tight, electrically insulating seal (see fig. 1(a)) that will have a life measured in years while subjected to high temperature and perhaps a severe radiation flux will be difficult. The gas cooled or heated diode is not considered further in this analysis.

Indirect Cooling of Anodes

These considerations indicate that a scheme in which the heattransfer agent does not contact the electrodes has a better chance of successful development. For such a system there would be no objection to liquid metals in regard to short circuiting, and the cathodes and the anodes could be maintained near the respective optimum temperatures. However, any indirect heat-transfer scheme would exact a penalty through additional temperature drops.

Two possible techniques for achieving such a design are considered in this study. Because it was concluded early in the study that the system using diodes integral with the reactor fuel elements held the most promise in comparison with a turbogenerator system, the discussion here is restricted to means of cooling the anode indirectly. One of the ways is to use radiation from the anodes to a wall cooled by a liquid metal. In cylindrical geometry this design would take the form of a cylindrical capsule enclosing a stack of series-connected diodes cooled on the outside by a liquid metal (see fig. 1(b)). Spacing could be maintained by insulators, but there would be no need for insulating seals except on the end of the capsule, where provision for cooling of the seal region could be provided.

Another method is to use thermal conduction through an electrically insulating material between the anodes and the coolant wall (see fig. l(c)). Besides the temperature drop between the anode and the coolant there would be in this design a loss due to the leakage of current through the insulator which in some parts of the system would be subjected to hundreds of volts.

Temperature Limits

Before the diode system can be compared with the turbogenerator system, the factors that limit the temperatures of the systems must be considered. Corrosion, including mass transfer, by the liquid metal is an important factor in limiting the temperatures in both systems. The mass transfer problems in the two systems will be somewhat different because in the turbogenerator system two phases and large temperature differences will be present, while in the diode system the metal coolant loop will be all liquid and will not have as large temperature differences. Nevertheless, the maximum liquid metal temperature is assumed here to be the same for both systems.

Another factor in the turbogenerator system is high temperature creep of the turbine blades, but it is assumed not to be limiting in this analysis. The creep problem often can be alleviated if a sufficiently heavy and complex turbine is used. Since the turbine weight is a small

portion of the total weight of the powerplant, large concessions in turbine weight can be tolerated by the system to overcome the creep problem.

In the system employing diodes with combined fuel elements and cathodes the maximum radiator temperature will be set by corrosion, and the top temperature will be set by the strength of the fuel element material, the volatility of the cathode material and perhaps the properties of the insulators.

PERFORMANCE OF RANKINE CYCLE TURBOGENERATOR

The diode systems are compared in this study with kankine cycle turbogenerator systems on the basis of radiator area. The results of reference 2 were used in this study to compute the minimum radiator area required for a turbogenerator with a turbine inlet temperature of 2000° F for turbine efficiencies of 0.80, 0.70, and 0.60. The emissivity of the radiator surface was taken as 0.90. The turbine efficiency can be interpreted as including the mechanical and electrical generating losses as well as those in the turbine itself, and henceforth $\eta_{\rm t}$ will be called the conversion efficiency. Sodium was assumed as the working fluid, but the radiator area required does not depend very much on which alkali metal is selected. The following table gives the cycle temperature ratios, efficiencies, and specific radiator area for the prescribed conditions:

η_{t}	$T_{\rm R}/T_{\rm ti}$	$\eta_{ ext{cycle}}$	A _R /P, ft ² /kw
0.8	0.764	0.178	0.800
.7	.769	.151	.952
•6	.775	.127	1.134

Radiator areas at higher temperatures were assumed proportional to $T_{\rm ti}^{-4}$. Because it is anticipated that a turbine inlet temperature of 2000° F will be attainable by the time a thermionic system will be ready for development, radiator areas for lower temperatures were not computed.

DERIVATION OF EQUATIONS

This analysis is made for a system of parallel flat plate diodes connected in series. Equations are written and performance is computed for a typical diode in this system. The thermionic current emission and power generation are computed in the same way as in reference 1. The height of the potential barrier above the Fermi levels in the cathode and the anode, $V_{\rm C}$ and $V_{\rm A}$, respectively, are used as parameters. It is

assumed that electrode spacing and/or interelectrode gas density is low enough that electron-ion and electron-electron collisions can be neglected. Thermionic emission from the anode is included. The losses considered are joulean heating in the electrodes and the series conductor, electron heating of the anode, heat radiation from the cathode to the anode, and heat conduction through the conductor from cathode to anode. The effect on electron emission due to the resistive voltage drop along the electrodes is ignored. Several ways of cooling the anode are considered.

Current

The density of current emitted from each electrode is given by the Richardson-Dushman equation, which for the cathode is

$$J_{C} = \mathcal{J}_{RD} T_{C}^{2} \exp \left(-\frac{eV_{C}}{kT_{C}}\right)$$

and for the anode

$$J_{A} = \mathcal{A}_{RD} T_{A}^{2} \exp \left(-\frac{e V_{A}}{k T_{A}}\right)$$

All the symbols are listed in appendix A and some are pictured in figure 2. The net current from cathode to anode is $J_n=J_C-J_A$.

Load Voltage

The power per unit of electrode area available for external work is $J_n V_O$, where $V_O = V_C - V_A$. The power usefully consumed in the load is less than this. If the load voltage is called V_L , the useful power is $J_n V_L = J_n V_O$ - Dissipative losses in the circuit. The dissipative losses that will be considered here are the joulean heating in the electrodes and in the conductors connecting adjacent elements in series. The power loss per unit area in the electrodes is $\frac{1}{3} \frac{T^2}{A_E} \left(R_C + R_A \right) = \frac{1}{3} J_n^2 A_E \left(R_C + R_A \right)$. In the conductor the loss is $J_n E_{CON}$. If both electrodes are assumed to have the same resistivity and size, $R_C + R_A = 2 \frac{\rho z}{wt} = 2 \frac{\rho z^2}{A_E t}$, and the expression for V_L becomes

$$V_{L} = V_{o} - \frac{2}{3} J_{n} \rho \frac{z^{2}}{t} - E_{con}$$
 (1)

The resistivity of the electrodes and the conductor was taken to be 50 microhm centimeter, a value typical of the refractory metals in the temperature range considered.

Heat Rejected

Besides these dissipative losses there are the heat-transfer losses that compose the heat that must be rejected from the anode. The heat transferred includes the radiation between cathode and anode, the kinetic energy carried by the electrons entering and leaving the anode surface, and the heat conducted to the anode through the electrical leads. It is assumed that the equation for gray-body radiation adequately describes the actual process. For infinite parallel gray plates the net heat radiated is given by

$$\frac{Q_{C}}{A_{E}} = \frac{5.77}{\frac{1}{\epsilon_{C}} + \frac{1}{\epsilon_{A}} - 1} \left[\left(\frac{T_{C}}{1000} \right)^{4} - \left(\frac{T_{A}}{1000} \right)^{4} \right] = \frac{5.77}{\frac{2}{\epsilon_{CA}} - 1} \left[\left(\frac{T_{C}}{1000} \right)^{4} - \left(\frac{T_{A}}{1000} \right)^{4} \right] (2)$$

where ϵ_{CA} is an effective emissivity, $\epsilon_{CA} = \frac{2}{\frac{1}{\epsilon_{C}} + \frac{1}{\epsilon_{A}}}$. The value of

 ϵ_{CA} used was 0.22, which was considered typical of refractory metal electrodes at the temperatures considered. The heating of the anode by an electron reaching it from the cathode results from the kinetic energy given up by the electron as it reaches equilibrium in the anode. This kinetic energy is the sum of the kinetic energy the electron has as it crosses the peak of the space charge barrier (an average value of $2 \frac{k}{\epsilon} T_{\text{C}}$) plus the difference in potential energy between the peak and the anode Fermi level, that is, V_{A} . An electron emitted from the anode

that crosses the space charge barrier takes from the anode the energy equal to the height of the barrier V_A plus the kinetic energy it has as it crosses the peak, which on the average is $2 \frac{k}{e} T_A$. The net electron heating of the anode per unit area is

$$J_{C}\left(V_{A} + 2 \frac{k}{e} T_{C}\right) - J_{A}\left(V_{A} + 2 \frac{k}{e} T_{A}\right) \tag{3}$$

The heat conducted through the electrical leads to the anode is composed of the heat conducted because of the temperature difference across the conductor $T_C - T_A$ plus one-half of the joulean heating in the lead. (Radiation from the lead was not considered.) However, because of the nature of metals, the voltage drop in the leads and the heat conducted from cathode to anode are not independent. The relation between electrical resistivity and thermal conductivity is well described for metals by the Wiedemann-Franz law $\frac{KO}{T} = \mathcal{L}$, where \mathcal{L} is a constant. By means of this relation the expression for the heat conducted to the anode per unit cathode area as a function of the voltage E_{CON} is shown in appendix B to be

$$\frac{\mathbf{Q}_{con}}{\mathbf{A}_{E}} = \frac{0.015 \text{ J}_{n}}{\mathbf{E}_{con}} \left[\left(\frac{\mathbf{T}_{C}}{1000} \right)^{2} - \left(\frac{\mathbf{T}_{A}}{1000} \right)^{2} \right] + \frac{\mathbf{J}_{n} \mathbf{E}_{con}}{2}$$
(4)

Changing the geometry of the lead to reduce the voltage drop increases the heat conduction, and vice versa. There is an optimum compromise for these losses, and an expression for the value of $E_{\rm con}$ that yields the minimum radiator area per unit of output power is developed in appendix B. Combining equations (2) to (4) and the joulean heating in the anode $\frac{1}{3}\frac{J_{\rm n}^2\rho z^2}{t}$ gives for the heat per unit cathode area that must be rejected

$$\frac{Q_{R}}{A_{E}} = \frac{5.77}{\frac{2}{\epsilon_{CA}} - 1} \left[\left(\frac{T_{C}}{1000} \right)^{4} - \left(\frac{T_{A}}{1000} \right)^{4} \right] + \frac{1}{3} J_{n}^{2} \rho \frac{z^{2}}{t} + J_{n} V_{A} + 2k (T_{C} J_{C} - T_{A} J_{A})
+ \frac{0.015 J_{n}}{E_{COn}} \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right] + \frac{1}{2} J_{n} E_{COn}$$
(5)

where

$$E_{con} = \frac{0.015 \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right]}{\frac{V}{2} + \frac{q}{J_{n}}} \left\{ \sqrt{1 + \frac{V}{0.015 \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right]}} - 1 \right\}$$

Radiator Temperature

Three relations between anode temperature and radiator temperature are used. For some of the calculations the anode was assumed to radiate directly to space or to be cooled by an ideal heat-transfer loop, and the radiator temperature was assumed to be equal to the anode temperature. For other calculations radiative heat transfer from the anode to a coolant was presumed. Here also the coolant loop was taken to be ideal and the radiator temperature was set equal to the coolant temperature. Gray-body radiation was assumed between the anode and the coolant wall, and the coolant temperature was calculated from

$$\left(\frac{T_{R}}{1000}\right)^{4} = \left(\frac{T_{A}}{1000}\right)^{4} - \frac{Q_{R}}{A_{E}}\left(\frac{\frac{2}{\epsilon_{AW}} - 1}{5.77}\right)$$

A value of 0.8 was used for ϵ_{AW} .

A third case of cooling the anode by heat conduction through an electrical insulator to the coolant wall was also considered. A compromise between excessive temperature drop and excessive electrical leakage across the insulator must be made in setting the insulator thickness. The temperature drop is given by

$$T_{R} - T_{A} = \frac{Q_{R}}{A_{E}} \frac{d}{K}$$
 (6)

and the leakage power is

$$\Delta \frac{P}{A_E} = \frac{E^2}{\rho d} \tag{7}$$

An approximate formula for the thickness that allows the minimum specific radiator area (eq. (C7)) is developed in appendix C. This optimum thickness depends on operating conditions of the diode, the thermal conductivity and electrical resistivity of the insulator, and the voltage across the insulator E. In general this voltage would not be what has been termed here as the load voltage, or the voltage output per diode. It would be the voltage between the anode and the ground of the entire system and would depend on which anode in the series circuit is considered. In order to evaluate the effect on the system of the losses in the insulator an average voltage difference must be assumed, and the value taken for this study is 100 volts. Calculations were made for a range of insulator properties.

Radiator Area and Efficiency

The radiator area required to reject the heat is given by $A_R = \frac{Q_R}{\sigma_{SB} \varepsilon_R^{T_R^4}}.$ The radiator emissivity was assumed to be 0.9, the same

as for the turbogenerator system. Dividing by the output power $P=J_{\rm n}A_{\rm E}V_{\rm L}$ yields the expression for specific radiator area

$$\frac{A_{R}}{P} = \frac{Q_{R}/A_{E}}{\sigma_{SB} \epsilon_{R}^{T_{R}^{4}} J_{n}^{V} L}$$
(8)

The ratio of radiator area to electrode area is

$$\frac{A_{R}}{A_{E}} = \left(\frac{A_{R}}{P}\right) \left(\frac{P}{A_{E}}\right) = \frac{Q_{R}/A_{E}}{\sigma_{SB} \epsilon_{R}^{T_{R}^{4}}}$$

The efficiency is given by

$$\eta = \frac{P}{P + Q_R} = \frac{1}{1 + \frac{Q_R/A_E}{J_n V_L}}$$
(9)

APPLICATION OF THE EQUATIONS

With the equations derived in the previous section, $J_{\rm n},\ V_{\rm L},\ T_{\rm R},$ $A_{
m R}/P$, η , and $A_{
m R}/A_{
m E}$ can be computed after the properties of the materials and T_C , T_A , V_C^- , and V_A are specified. The prime interest in this study is A_R/P. For fixed values of $T_{\rm C}$, $V_{\rm C}$, and $V_{\rm A}$ the variation of $A_{
m R}/{
m P}$ and $T_{
m R}$ with $T_{
m A}$ can be computed. Typical curves of $A_{
m R}/{
m P}$ against T_{R} for fixed V_{C} and V_{A} are shown dashed in figure 3. dashed curve exhibits a clear minimum where the best compromise between excessive back emission and low radiator temperature exists. Only the minimum point on each curve will be considered further. Once this restriction is made it is impossible to plot an operating line (a curve for a fixed geometry and prescribed material properties) on this chart, and it is important that none of the curves be so construed. Families of curves for constant $V_{\mathbb{C}}$ and for constant $V_{\mathbb{A}}$ can be computed, and each family can be characterized by a curve through the minima, as shown in figure 3. It develops that at each value of $\ensuremath{T_{R}}$ there is a combination of V_C and V_A that gives a minimum value of A_R/P . It further develops that there is a value of $T_{\mbox{\scriptsize R}}$ for which this minimum $A_{\mbox{\scriptsize R}}/P$ is lower than for all other values of $T_{\rm R}$, and this value of $A_{\rm R}/P$ is the very best possible for the specified cathode temperature and diode configuration.

The use of the loci of the minima as just described requires some discussion. The envelope of these loci does not give the true envelope of solutions except where the slope of the envelope is zero. Fortunately in this study the slope of the envelope is small in the range of interest, and there is little error in using the loci of the minima. This was verified by calculations. Because the slope of curves of constant $V_{\mathbf{C}}$ is small, they may be used as good approximations to the envelope of solutions with that value of Vc. These curves indicate the best $A_{
m R}/{
m P}$ possible when $T_{
m R}$ and $V_{
m C}$ are given. The slope of the curves of constant $V_{\rm A}$ is small only near the minimum. Away from the minimum, where the locus differs from the envelope substantially, these curves do not indicate the optimum $A_{\mathbb{R}}/P$ for fixed $V_{\mathbb{A}}$. They simply show what value of $V_{\rm A}$ must exist in order that for a given $V_{\rm C}$ and $T_{\rm R}$ the optimum A_R/P will be achieved. At a given $T_{\rm R}$ and $V_{\rm A}$ there will be values of $A_{\rm R}/{\rm P}$ less than shown by the loci of the minima (see fig. 3).

RESULTS

Direct and Ideal Cooling of Anode

The calculated performance for diode systems in which the radiator temperature is equal to the anode temperature is presented in detail. These results apply to systems in which the anodes radiate directly to space. They are also significant as the limit of performance for systems in which there is a heat-transfer circuit between the anodes and the radiator.

The comparison of the diode and turbogenerator systems is made in two ways. Both methods are based on the assumption that the liquid metal temperature is a limiting factor. The first compares systems with the same peak temperature and applies to diode systems in which the cathode is heated by a liquid metal. In the other method the diode system is compared with a turbogenerator which is limited to a turbine inlet temperature equal to the radiator temperature of the diode powerplant. This comparison applies to diode systems in which the cathodes are integral with the fuel elements and the anodes are cooled by a liquid metal loop to the radiator.

General character of diode performance. - Curves of $A_{\mbox{\scriptsize R}}/P$ against T_R for $T_R = T_A$ are presented for various values of V_C and V_A and for several values of $T_{\mathbb{C}}$ in figure 4. The electrode thickness parameter z^2/t is equal to 125 centimeters. (The turbogenerator curves are irrelevant to the present discussion.) At each cathode temperature these curves have an envelope which indicates the minimum possible radiator area at each anode temperature. The value of $\,{\mbox{V}}_{\mbox{\scriptsize C}}\,\,$ is nearly the same for the whole envelope. Curves of fixed $V_{\mathbb{C}}$ less than this optimum value and the portions of the curves of fixed $\,V_{\hbox{\scriptsize A}}\,\,$ which correspond to these low values of V_{C} have been excluded. This envelope exhibits a minimum when the ratio of anode to cathode temperature is near 0.76, and the shape of the curve when plotted against T_A/T_C is the same for all the cathode temperatures and electrode thicknesses. For a cathode temperature of 1900° K and $z^2/t = 125$ values of A_R/P within 10 percent of the minimum are possible with values of V_{Δ} between 1.7 and 2.3 volts and values of V_C between 2.5 and 2.8 volts. Curves of V_C = 3.2 and $V_A = 2.5$ indicate how sensitive the radiator area is to these parameters. The half-volt increase in $\,V_{C}\,$ from the optimum value results in more than a doubling of the radiator area; the half-volt increase in V_{Δ} increases the minimum possible area about 30 percent.

Several values of z^2/t were considered in this study. The following table lists the values of z^2/t used and the lengths and thicknesses which can correspond to these values:

z^2/t , cm	t, cm		
	z = 5 cm	z = 2.5 cm	
250	0.1	0.025	
125	.2	.050	
50	•5	.125	
0	∞	∞	

The effect of z^2/t on the envelope is shown in figure 5. The fractional change in A_R/P between the curves for a fixed T_C is independent of T_A . The ratio of A_R/P with electrode loss to that without electrode loss is shown in figure 6.

The value of z^2/t that would be selected by a designer would be influenced by factors besides the performance computed here. The number of diodes needed to furnish a given total power and end effects on diode performance place a lower limit on the length of the electrodes. The mass of the electrodes, through its direct contribution to the total mass of the system and the effect of neutron poisoning on the design of the reactor with integral fuel elements and diodes, restricts the thickness of the electrodes. Figure 6 indicates that with respect to radiator area there is not much to be gained in decreasing z^2/t below 50.

Figure 7 summarizes the conditions that yield minimum radiator area. Figure 7(a) shows the value of V_C along the minimum A_R/P envelope as a function of T_C and z^2/t . Figure 7(b) shows the variation of V_A along the envelope with T_A and z^2/t . This V_A depends very little on T_C . The widths of the bands for $z^2/t = 0$ and 50 indicate the variation due to the range in T_C . Similar bands for $z^2/t = 125$ and 250 would overlap, and they have been narrowed in the figure for clarity.

These results give some idea of the electrode work functions needed for low-radiator-area designs because $\phi_C \leq V_C$ and $\phi_A \leq V_A$. Work functions in the range shown in figure 7 are possible with tungsten electrodes and cesium vapor at a pressure in the range of several millimeters of mercury. With uranium monocarbide, whose work function is about 3 volts, lower cesium pressures would be adequate. But with these low work functions few ions can be formed by contact ionization. Ionization by electron-ion collisions may be necessary. The losses associated with such collisions and the heat conduction through the high-pressure cesium vapor, which were neglected in this analysis, may be large.

Curves of constant load voltage are presented in figure 8 for cathode temperatures of 1900° and 2300° K and values of z^2/t of 50, 125, and 250 centimeters. Only the portions of the curves for $V_{\rm C}$ greater than the optimum are shown. For all the cathode temperatures and electrode thicknesses considered the load voltage along the minimum area envelope depends principally on the temperature difference between the cathode and the anode. Load voltages in excess of 1 volt are possible with temperature differences greater than 800° K.

Shown in figure 9 are curves of constant efficiency and curves corresponding to Carnot cycles for several peak temperatures in a plot of A_R/P against T_R . With the radiator emissivity fixed, A_R/P is a function of T_R and η ; specifically,

$$\frac{A_R}{P} = \frac{1 - \eta}{\sigma_{SB} \epsilon_R T_R^4 \eta}$$

Therefore these curves of constant efficiency can be laid over the plot of $A_{\rm R}/P$ against $T_{\rm R}$ for any cycle with this emissivity.

Comparison of figures 5 and 9 indicates that at the minimum radiator area point the efficiency varies from 0.12 for $T_C=2300^{\circ}~K$ and $z^2/t=250$ centimeters to 0.16 for $T_C=1700^{\circ}~K$ and $z^2/t=0$. For the minimum radiator area point T_R/T_C is about 0.76 and the Carnot efficiency is 0.24. The diode efficiency at this point is 50 to 65 percent of the Carnot efficiency. The ratio of diode efficiency to Carnot efficiency is about the same at the lower anode temperatures as at the minimum area point.

In general radiator-to-cathode temperature ratios greater than 0.76 are undesirable. Load voltage, power density, and efficiency decrease as the temperature ratio increases. Above 0.76 the radiator area increases also, and there is no reason to design in this range.

Diodes mounted in radiator. - Curves of fixed ratio of radiator area to electrode area are shown in figure 10. Values of this ratio greater than unity indicate designs that require fins on the anode to reject enough heat. Values less than unity indicate impractical designs and have been excluded. The portion of each curve to the right of its minimum corresponds to values of $\rm V_C$ less than the optimum and is not shown. Figure 10 shows that designs with minimum radiator area are possible with reasonable values of area augmentation, say $\rm A_R/\rm A_E$ up to 2.5.

The variation of minimum specific radiator area with cathode temperature is shown in figure 11 and is compared with that for a Rankine cycle turbogenerator with a turbine inlet temperature equal to the cathode

temperature. The radiator of the thermionic system is 25 to 50 percent larger than that of a turbogenerator with a conversion efficiency of 0.8. The diode system must be able to operate 100° to 200° K hotter than this very efficient turbogenerator to use the same radiator size. The diode system can be competitive with the turbogenerator with the same peak temperature if turbogenerator conversion efficiencies of 0.7 cannot be attained.

Diodes mounted in reactor. - Diodes that are integral with the reactor fuel elements should be compared with a turbogenerator with a turbine inlet temperature equal to the radiator temperature of the thermionic powerplant. The radiator area for such turbogenerators is shown with the thermionic requirements when $T_R = T_A$ in figure 4. This comparison with $T_R = T_A$ does not take into account any temperature drop for cooling the anodes without short circuiting them. Nevertheless the comparison has value in that it indicates the upper limit of the gains possible. To facilitate the comparison the envelopes of the thermionic diode curves are plotted in figure 5 with the turbogenerator curves. The ultimate potential of this fuel-element - diode system is indeed substantial. Radiators one-fifth the size of those for turbogenerators are possible if the cathode temperature can go to 2300° K. Even at 1700° K the thermionic system requires only half the radiator area for the turbogenerator.

Indirect Cooling of Anode

It is shown in the previous section that an ideally cooled diode integral with the fuel element can have a radiator area substantially smaller than a turbogenerator. In this section the effect of cooling the anode by radiation and by conduction through an electrical insulator is investigated.

Diodes cooled by radiation. - The minimum radiator area requirements for the radiation-cooled diode system with $z^2/t=125$ centimeters are shown in figure 12 for several cathode temperatures. This system optimized with very low current and power densities and was insensitive to z^2/t . There is some improvement possible in radiator requirements over the turbogenerator, but it is shown in the section Diode Power Density and Reactor Size that the low power density requires an overly large reactor.

Diodes cooled by conduction through insulator. - In appendix C it is shown that the losses in an insulator sized for minimum radiator area depend on the product ρK . Calculations of A_R/P were made over a range of ρK for cathode temperatures of 1900° and 2300° K. The results are summarized in figure 13, where the ratio of the minimum

 A_R/P with the optimum insulator to the minimum A_R/P with no insulator loss is presented as a function of ρK . For high values of ρK the radiator area is not very sensitive to ρK . Under these conditions of low insulator loss all the results for $T_R=T_A$ remain essentially the same.

However, at values of ρK less than about 5×10^3 (w)(ohms)/ $^{\circ} K$, A_R/P is seriously affected by the insulator properties. It appears that values of ρK greater than this may be possible for this application. References 3 and 4 indicate thermal conductivities of about 0.06 w/(cm)($^{\circ} K$) for aluminum oxide and about 0.12 w/(cm)($^{\circ} K$) for beryllium oxide in the temperature range 1200° to $1600^{\circ} K$. References 3 and 5 indicate a considerable spread in the measured values of electrical resistivity for aluminum oxide. At $1200^{\circ} K$ the measured values range from 10^6 to 10^8 (ohms)(cm) and at $1600^{\circ} K$ from 10^3 to 10^5 (ohms)(cm). Reference 4 gives values for beryllium oxide near the top of this range. Except for the lower values for aluminum oxide at the higher temperatures the measured ρK for these materials is greater than $5 \times 10^3 (w)(ohms)/{^{\circ}} K$. These measured values are for unirradiated specimens. What effect the intense irradiation in the reactor core will have on these properties is unknown.

Diode Power Density and Reactor Size

If the power density of the diode, that is, the watts of output power generated per square centimeter of cathode area, is low, the reactor core size needed to contain electrode area sufficient to supply the prescribed total power may be higher than that set by nuclear or heat-transfer limitations. The power density of the diode can seriously affect the mass of the entire system if it is so low that it influences the reactor size and concomitantly the weight of the nuclear shield required.

Reactor diameter. - In the most attractive design for mounting diodes in a reactor, a dozen or two diodes, together with the nuclear fuel in the cathodes, are assembled into tubular elements. These elements are packed together to make up the core of the reactor. Liquid metal circulates among the tubes and cools each externally. The most compact pattern for packing the tubes is hexagonal, and for an infinite array of tightly packed tubes the volume external to the tubes is 9.3 percent of the total. An equation is developed in appendix D (eq. (D4)) for the diameter of a cylindrical core with the same volume as that of the packed-tube core. Figure 14 shows the reactor diameter required to hold diodes capable of furnishing l megawatt of power as a function of diode power density. Assumed in this plot are reactor length-diameter ratio of unity, coolant volume fraction of 0.1, tube diameter of 1 inch, cathode diameter of 0.7 inch, and a 10-percent addition to the length

of the tubes as an allowance for spacers between diodes. Also shown in this plot are reactor sizes needed for a 1-megawatt turbogenerator system (data received from R. D. Brooks of General Electric Co.). These reactor sizes for the turbogenerator system are determined by nuclear criticality and heat-transfer requirements. A thermionic diode reactor cannot be made as small as the very compact fast reactor for the turbogenerator system because of the dilution and neutron poisoning of the core by diode structure and voids. For the size of the diode reactor not to exceed that of the intermediate spectrum reactor the diode power density must be at least 10 watts per square centimeter.

Power density. - The values of power density to be presented here are those for conditions that give minimum specific radiator area for fixed V_{C} and V_{A} , and they are often far from the maximum possible power density. Curves of specific radiator area against power density for several cathode temperatures are presented in figure 15 for the case where T_{R} equals T_{A} . The significant characteristic of these curves is that there is a wide range of power densities possible with little change in radiator area for values of V_{C} near the optimum.

The variation of power density with z^2/t for several values of T_C and V_A is presented in figure 16. For low values of V_A the electrode thickness has a strong effect on the power density of a diode optimized for low radiator area.

The power density for diodes cooled through an insulator is essentially the same as that just presented if the losses in the insulator are low. If low anode work functions can be attained for this application, reactor size will not be limited by the geometrical considerations of fitting sufficient diodes into the core. In contrast, the radiation-cooled diodes, which were only marginally better than the turbogenerator with regard to radiator area, optimized at power densities less than 2 watts per square centimeter and would require a very large reactor.

Of course, power densities higher than the geometrically limiting value will be desirable so that fewer diodes will have to be fitted into the core. Shortening the electrodes can help, but not thickening. For fixed electrode length z and fixed total power, the electrode volume is proportional to $1/p(z^2/t)$. The information in figure 16 is replotted in figure 17 as $1/p(z^2/t)$ against z^2/t . As z^2/t decreases, corresponding to an increasing t, the electrode volume increases. The higher dilution of the core will require a larger diameter to satisfy criticality.

DISCUSSION

This preliminary comparison of the thermionic and turbogenerator systems was necessarily based on simplified ground rules that (1) radiator area is a good index of mass, and (2) liquid metal corrosion limits the temperature and this limit is the same in both systems. Such an analysis is useful only in identifying large differences between the systems. This analysis shows that (1) there is theoretically possible a large saving in radiator area if reactor mounted diodes cooled through an insulator are used, and (2) there is little or no saving with the other diode schemes. Further effort on the reactor mounted system is certainly in order.

Of course, much depends on whether diode performance near the optimum can actually be achieved. Important factors are the work functions of the electrodes, the strength and volatility of the electrodes, the electrode spacing, and properties of the insulators. But there are other factors apart from diode performance that will influence the final selection of a system and may invalidate the two ground rules.

Radiator area may not be a good index of system mass because:

- (1) Radiator mass may be far different for the same area in the two systems.
- (2) Turbine and generator mass may be more significant than anticipated.
- (3) The thermionic reactor may be much larger because of voids and poisoning. The corresponding increase in shield mass may overshadow all other differences between the systems.

The two systems may not be limited to the same liquid metal temperature because

- (1) Turbine blade creep and erosion may limit the turbogenerator.
- (2) The corrosion problem may be much different in the two systems. Probably the systems will use different liquid metals.
- (3) Fuel element and electrode properties such as creep and volatility or insulator properties may limit the temperatures in the thermionic system.

SUMMARY OF RESULTS

The results of a parametric analysis of nuclear-electric powerplants using thermionic diodes in which the required radiator area was the index of performance are as follows:

- 1. For each cathode temperature there is an optimum height of the potential barrier above the Fermi levels in the cathode and the anode that permits minimum radiator area.
- 2. At a fixed cathode temperature the radiator-to-cathode temperature ratio for this minimum radiator area is near 0.76.
- 3. Electrode length and thickness have an important effect on radiator area and power density.
- 4. Increasing electrode thickness increases power density but not enough to decrease the volume of electrodes needed for a given total power.
- 5. A turbogenerator with a conversion efficiency of 0.7 requires less radiator area than thermionic diodes operating with the same peak temperature.
- 6. A thermionic diode system in which the anodes are cooled by radiation to a liquid metal coolant circulating through an external radiator requires about the same radiator area as a turbogenerator system with a turbine inlet temperature equal to the radiator temperature of the diode system.
- 7. A large reactor core would be needed to house radiation-cooled diodes because the power density possible with a design for low radiator area is very low.
- 8. Diodes mounted in the reactor and cooled by conduction through electrical insulators show potentially high savings in radiator area (up to 80 percent at a cathode temperature of 2300° K) when compared with a turbogenerator system with a turbine inlet temperature equal to the radiator temperature of the diode system.

Lewis Research Center

National Aeronautics and Space Administration Cleveland, Ohio, September 12, 1961

APPENDIX A

SYMBOLS

 $A_{\rm E}$ electrode area, cm²

 A_{R} radiator area, sq ft

 \mathcal{A}_{RD} Richardson-Dushman coefficient, 120.1 amp/(cm²/(°K)²)

a conductor cross-sectional area, cm²

 $D_{\mathbf{C}}$ cathode diameter, cm

D_{RC} reactor core diameter, in.

 D_{rp} tube diameter, in.

d insulator thickness, cm

E average anode to ground voltage, v

 \mathbf{E}_{con} —voltage drop through conductor between anode and next cathode, \mathbf{v}

e electronic charge, 1.60×10⁻¹⁹ coulomb

I current, amp

J current density emitted from anode, amp/cm²

 J_C current density emitted from cathode, amp/cm²

 J_n net current density, amp/cm^2

j leakage current density, amp/cm²

K thermal conductivity, w/(cm)(°K)

k Boltzmann constant, 1.38×10⁻²³ joule/oK

 L_{C} length of cathode surface in one tube, in.

 ${f L}_{
m RC}$ reactor core length, in.

 \mathscr{L} constant in Wiedemann-Franz law, $(w)(ohms)(oK)^2$

l conductor length, cm

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N number of tubes
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- P power, w
- p diode power density, w/cm²
- $Q_{\mathbf{C}}$ heat radiated from cathode to anode, w
- heat conducted from conductor to anode, w
- Q_R heat rejected from radiator, w
- q term defined in appendix B
- R_{Δ} anode resistance, ohms
- R_C cathode resistance, ohms
- T temperature, ^OK
- T_{A} anode temperature, ${}^{\mathsf{O}} K$
- T_C cathode temperature, ^OK
- T_R radiator temperature, ^OK
- T_{ti} turbine inlet temperature, ${}^{\circ}K$
- t thickness of electrodes, cm
- V_{Δ} height of potential barrier above anode Fermi level, v
- $V_{\mathbf{C}}$ height of potential barrier above cathode Fermi level, v
- v_{τ} voltage output per diode
- v_{o} v_{c} v_{A}
- y reactor core volume, cu in.
- $_{
 m V}$ term defined in appendix B
- w width of electrodes, cm
- z length of electrodes, cm
- α fraction of reactor core volume filled with coolant

	$\epsilon_{ m A}$	anode emissivity
	ϵ_{AW}	average emissivity between anode and ccolant wall
	$\epsilon_{ extsf{C}}$	cathode emissivity
	$\epsilon_{ ext{CA}}$	average emissivity between cathode and anode
119	ϵ_{R}	radiator emissivity
E-1113	η	cycle efficiency
	η_{t}	turbine efficiency
	ρ	electrical resistivity, (ohms)(cm)
	σ	electrical conductivity, $1/\rho$
	σ_{SB}	Stefan-Boltzmann constant, 5.77×10 ⁻¹² $\text{w/(cm}^2)(^{\circ}\text{K})^4$, or 5.37×10 ⁻¹² $\text{kw/(ft}^2)(^{\circ}\text{K})^4$
. -	$\phi_{\!A}$	anode work function, v
	$\phi_{\textbf{C}}$	cathode work function, v

APPENDIX B

OPTIMUM VOLTAGE DROP IN CONDUCTOR

The expression for the heat conducted through the conductor to the anode in terms of the conductor voltage drop $E_{\rm con}$ is derived first, and the value of $E_{\rm con}$ that minimizes $A_{\rm R}/P$ is then determined.

The Wiedemann-Franz law relates quite reliably the thermal and electrical conductivity of metals as follows:

$$\frac{K}{\sigma T} = \mathcal{L} \tag{B1}$$

where \mathcal{L} is a constant, of which the theoretical value is 2.5×10⁻⁸ (w)(ohm)/°K². The values of K/oT for tungsten and tantalum are presented plotted against temperature in figure 18. A value of 3×10⁻⁸ (w)(ohm)/°K² was taken as typical for the refractory metals.

The heat conducted from cathode to anode per unit cathode area is given by

$$\mathbf{2} = \frac{\mathrm{Ka}}{\mathrm{A_E}} \frac{\mathrm{T_C} - \mathrm{T_A}}{\mathrm{1}}$$
 (B2)

where K is an average conductivity and will be taken here as the value at $T = \frac{1}{2} (T_C + T_A)$. The voltage drop in the conductor is given by

$$E_{con} = IR = J_n A_E \frac{l}{\sigma a}$$

where σ is an average conductivity and is taken as that at the same average temperature. If the average conductivities are related by equation (B1), equation (B2) becomes

$$\mathbf{Q} = J_n \frac{\mathbf{g}}{E_{con}} \frac{T_C^2 - T_A^2}{2}$$

Substitution of $\mathcal{L} = 3 \times 10^{-8}$ gives

$$\mathbf{2} = \frac{0.015 \text{ J}_{\text{n}}}{\text{E}_{\text{con}}} \left[\left(\frac{\text{T}_{\text{C}}}{1000} \right)^2 - \left(\frac{\text{T}_{\text{A}}}{1000} \right)^2 \right]$$

Adding half of the heat generated in the conductor gives the total heat transferred to the anode from the conductor per unit cathode area:

$$\frac{Q_{\text{con}}}{A_{\text{E}}} = \frac{0.015 \text{ J}_{\text{n}}}{E_{\text{con}}} \left[\left(\frac{T_{\text{C}}}{1000} \right)^{2} - \left(\frac{T_{\text{A}}}{1000} \right)^{2} \right] + \frac{1}{2} \text{ J}_{\text{n}} E_{\text{con}}$$
 (B3)

Combining the terms in equation (5) that are independent of $E_{\rm con}$ into q and those in (1) into v gives

$$\frac{Q_{R}}{A_{E}} = q + \frac{0.015 J_{n}}{E_{con}} \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right] + \frac{1}{2} J_{n} E_{con}$$
 (B4)

and

$$V_{L} = v - E_{con}$$
 (B5)

where

$$q = \frac{Q_C}{A_E} + \frac{1}{3} J_n^2 \rho \frac{z^2}{t} + J_n V_A + 2k(T_C J_C - T_A J_A)$$

and

$$v = V_C - V_A - \frac{2}{3} J_n \rho \frac{z^2}{t}$$

The optimum value of $E_{\mathbb{C}}$ is that for which

$$\frac{\partial E_{COD}}{\partial (A_{P})} = 0$$

or

$$\frac{1}{Q_{R}/A_{E}} \frac{\partial (Q_{R}/A_{E})}{\partial E_{con}} = \frac{1}{V_{L}} \frac{\partial V_{L}}{\partial E_{con}}$$
(B6)

Performing the differentiation gives

$$\frac{\partial (Q_{R}/A_{E})}{\partial E_{con}} = -\frac{0.015 J_{n}}{E_{con}^{2}} \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right] + \frac{1}{2} J_{n}$$
(B7)

and

$$\frac{\partial E^{cou}}{\partial \Lambda^{\Gamma}} = -1 \tag{B8}$$

Substituting equations (B7) and (B8) into (B6) yields

$$\frac{-\frac{0.015 \text{ J}_{n}}{E_{\text{con}}^{2}} \left[\left(\frac{T_{\text{C}}}{1000} \right)^{2} - \left(\frac{T_{\text{A}}}{1000} \right)^{2} \right] + \frac{1}{2} \text{ J}_{n}}{q + \frac{0.015 \text{ J}_{n}}{E_{\text{con}}} \left[\left(\frac{T_{\text{C}}}{1000} \right)^{2} - \left(\frac{T_{\text{A}}}{1000} \right)^{2} \right] + \frac{1}{2} \text{ J}_{n} E_{\text{con}}} = \frac{-1}{v - E_{\text{con}}}$$

which when solved for E_{con} becomes

$$E_{con} = \frac{0.015 \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right]}{\frac{v}{2} + \frac{q}{J_{n}}} \left\{ 1 + \frac{v}{0.015 \left[\left(\frac{T_{C}}{1000} \right)^{2} - \left(\frac{T_{A}}{1000} \right)^{2} \right]} - 1 \right\}$$
(B9)

APPENDIX C

OPTIMUM THICKNESS OF INSULATOR BETWEEN ANODE AND COOLANT WALL

The specific radiator area can be given by

$$\frac{A_{R}}{P} = \frac{Q_{R}/A_{E}}{\sigma_{SR}\epsilon_{R}(T_{A} - \Delta T)^{4}(J_{n}V_{L} - jE)}$$
(C1)

and

$$\Delta T = \frac{Q_R}{A_E} \frac{d}{K}$$
 (C2)

$$j = \frac{E}{od}$$
 (C3)

Because in the cases of interest values of power leakage jE are small, they are not added to $Q_{\rm R}$, and $Q_{\rm R}$ can be considered independent of d. Logarithmic differentiation of equation (Cl) with respect to d and setting the derivative of $A_{\rm R}/P$ equal to zero to find the condition for minimum area yield

$$\frac{1}{A_{R}/P} \frac{\partial}{\partial d} \begin{pmatrix} A_{R} \\ P \end{pmatrix} = 4 \frac{\frac{\partial \triangle T}{\partial d}}{T_{A} - \triangle T} + \frac{\frac{\partial j}{\partial d}}{J_{n}V_{L} - jE} = 0$$
 (C4)

Differentiation of (C2) and (C3) and substitution in (C4) give

$$\frac{4}{\left(T_{A} - \frac{Q_{R}}{A_{E}} \frac{d}{K}\right)} \frac{Q_{R}/A_{E}}{K} = \frac{E^{2}}{\left(J_{n}V_{L} - \frac{E}{\rho d}\right)\rho d^{2}}$$
 (C5)

Solved for d, equation (C5) becomes

$$d = \frac{-\left(1 - \frac{4}{E^2}\right) \pm \sqrt{\left(1 - \frac{4}{E^2}\right)^2 + 16 \frac{J_n V_L \rho T_A K}{E^2 (Q_R / A_E)}}}{\frac{8J_n V_L \rho}{E^2}}$$
 (C6)

For the range of variables considered in this study it is sufficiently accurate to ignore the $\left(1-\frac{4}{E^2}\right)$ terms, and the result then is

$$d \simeq \frac{E}{2} \sqrt{\frac{T_{A}}{J_{n} V_{L}} \frac{K}{A_{E}}}$$
 (C7)

It is interesting to notice that, according to equations (C7), (C2), and (C3), ΔT and the leakage power are both inversely proportional to $\sqrt{\rho K}$. Since the degree of compacting of a particular insulating material will affect both ρ and K, but oppositely, the product will tend to remain the same. Therefore the optimum values of the losses ΔT and jE will depend mainly on the material itself and not on its density. Of course the value of the optimum thickness will depend on the density.

APPENDIX D

REACTOR CORE SIZE

The reactor configuration considered here is that with packed tubes (not necessarily tightly packed), which contain the fuel and the diodes, running the length of the core and surrounded by coolant. Although in practice these tubes would be arranged in a hexagonal pattern and the design would require an integral number of tubes and that all courses be filled, for simplicity these restrictions were ignored in this estimate of core size. The index of core size adopted here is the diameter of a cylindrical core with volume fractions of the constituents the same as for the array of tubes.

If $\,L_{\hbox{\scriptsize C}}\,$ is the total length occupied by cathodes in each of $\,{\rm N}\,$ tubes, the total cathode area is

$$A_{E} = N\pi D_{C} L_{C} = \frac{P}{p}$$
 (D1)

The volume occupied by tubes is N $\frac{\pi}{4}$ D_T²L_{RC}. The total core volume, including coolant, is

$$\mathscr{V} = \mathbb{N} \frac{\pi}{4} \frac{D_{\mathrm{T}}^{2} L_{\mathrm{RC}}}{1 - \alpha} \tag{D2}$$

Substitution for N from equation (D1) gives

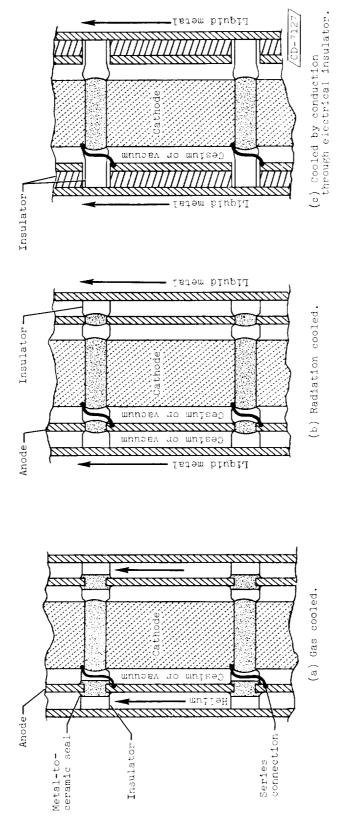
$$\mathbf{Y} = \frac{P}{P} \frac{D_{T}^{2} L_{RC}}{4D_{C} L_{C} (1 - \alpha)} \tag{D3}$$

For a reactor length-diameter ratio of unity the diameter of the equivalent cylinder is

$$D_{RC} = \left(\frac{4\pi}{\pi}\right)^{1/3} = \left(\frac{1}{\pi} \frac{P}{P} \frac{D_{T}^{2}}{D_{C}} \frac{L_{RC}}{L_{C}} \frac{1}{1-\alpha}\right)^{1/3}$$
 (D4)

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E-1119

Figure 1. - Configurations of thermionic diodes mounted in reactor core.

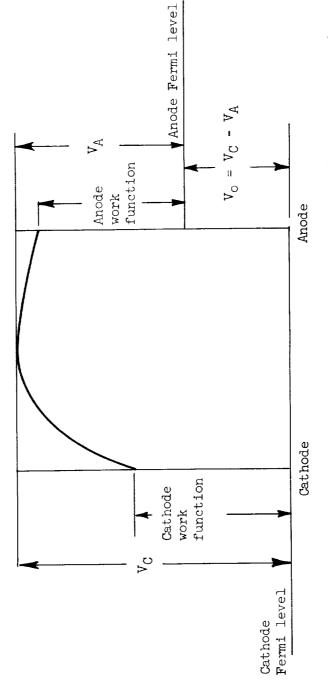
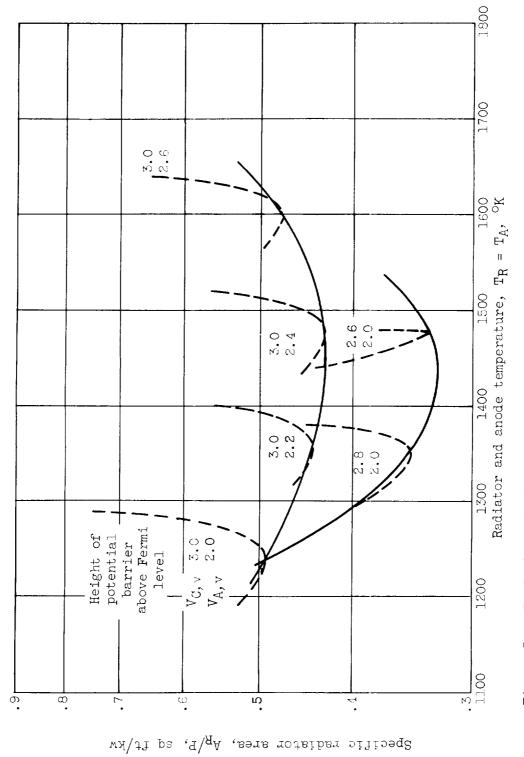


Figure 2. - Potential energy diagram for electrons in vacuum or low-pressure cesium thermionic diode.



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Figure 5. - Typical variations of specific radiator area with radiator temperature for fixed values of $\,V_{G}\,$ and $\,V_{A}.$

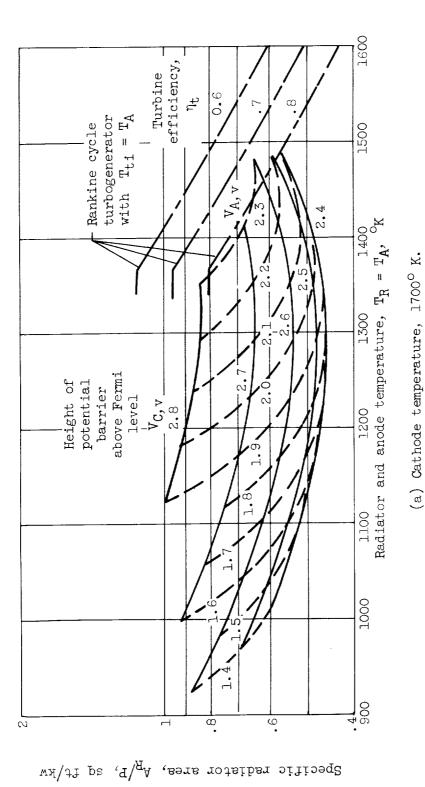


Figure 4. - Variation of specific radiator area with radiator temperature when radiator and anode temperatures are equal. Electrode thickness parameter, z^2/t , 125 centiand meters.



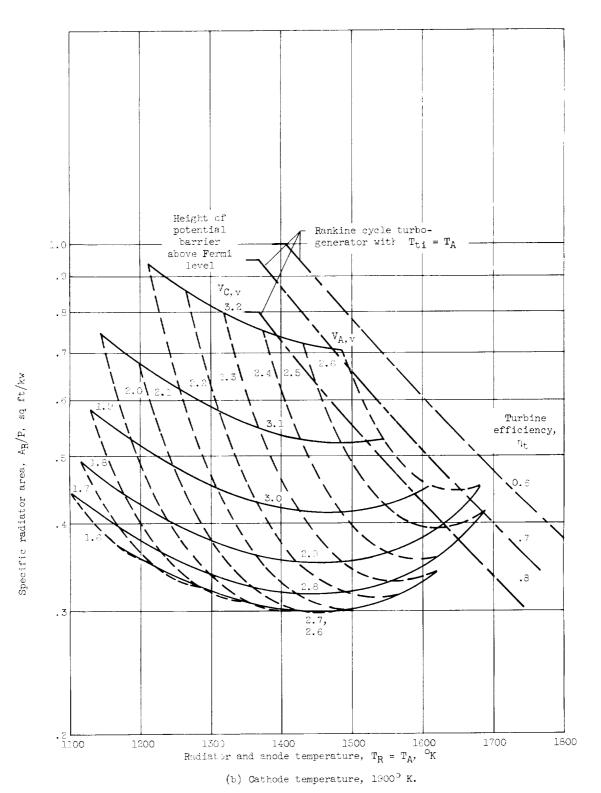


Figure 4. - Continued. Variation of specific radiator area with radiator temperature when radiator and anode temperatures are equal. Electrode thickness parameter, z^2/t , 125 centimeters.

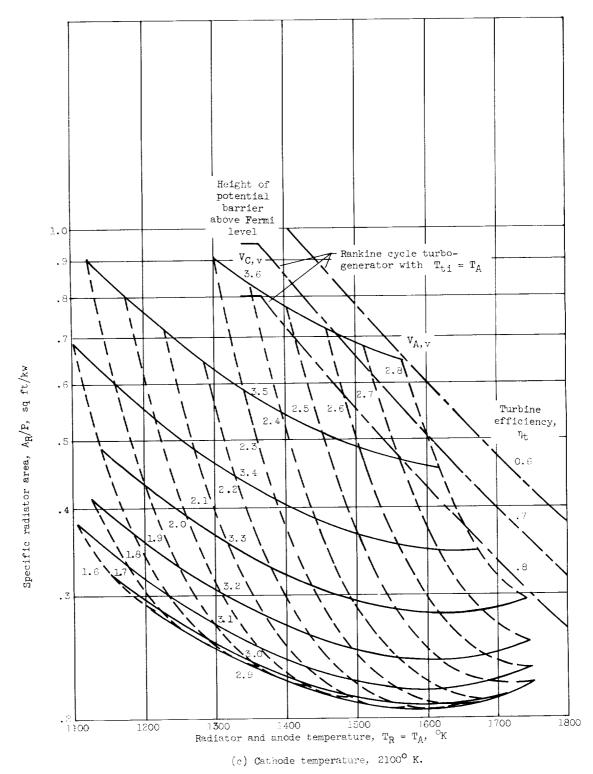


Figure 4. - Continued. Variation of specific radiator area with radiator temperature when radiator and anode temperatures are equal. Electrode thickness parameter, z^2/t , 125 centimeters.

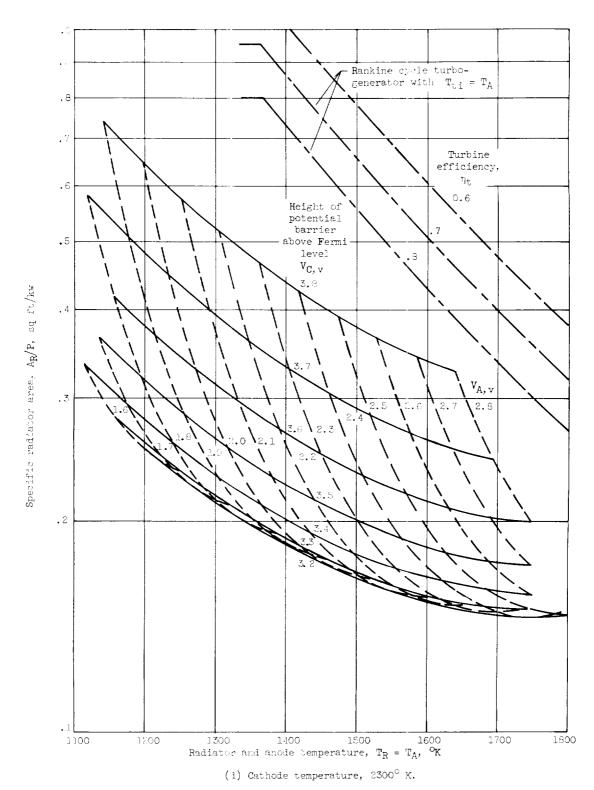


Figure 4. - Concluded. Variation of specific radiator area with radiator temperature when radiator and anode temperatures are equal. Electrode thickness parameter, z^2/t , 125 centimeters.

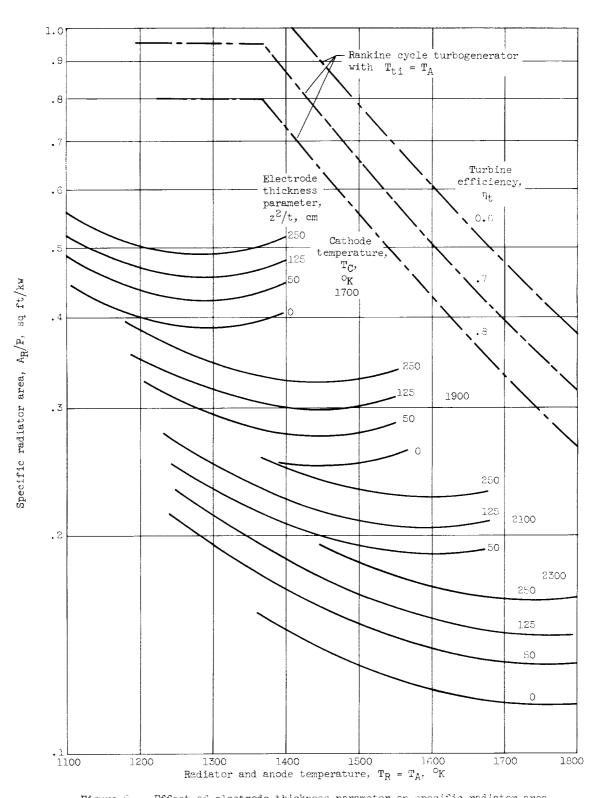
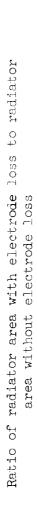
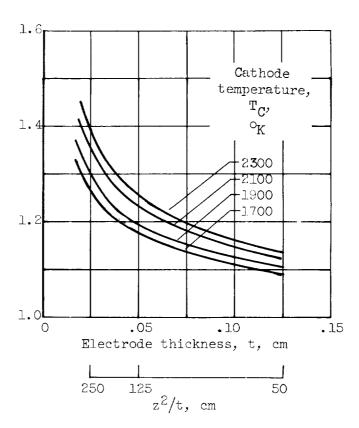
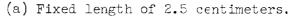
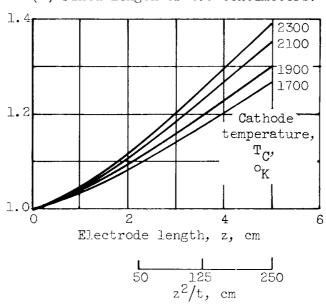


Figure 5. - Effect of electrode thickness parameter on specific radiator area.









(b) Fixed thickness of O.l centimeter.

Figure 6. - Effect of electrode length and thickness on radiator area.

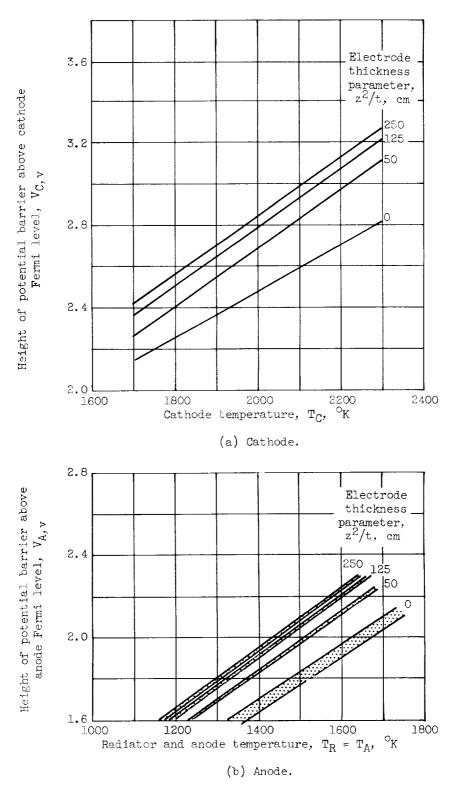


Figure 7. - Heights of potential barrier for minimum specific radiator area.

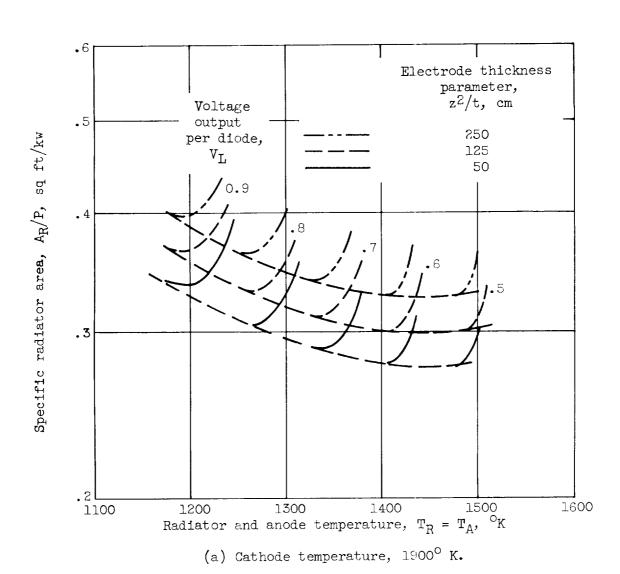


Figure 8. - Contours of fixed voltage output per diode.

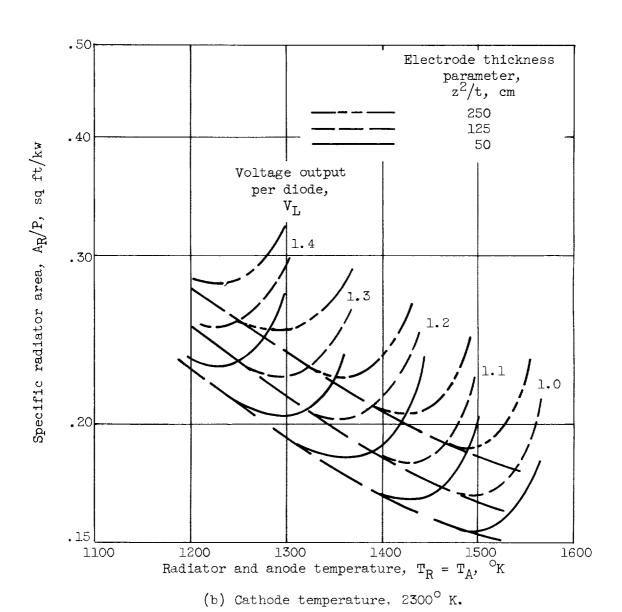


Figure 8. - Concluded. Contours of fixed voltage output per diode.

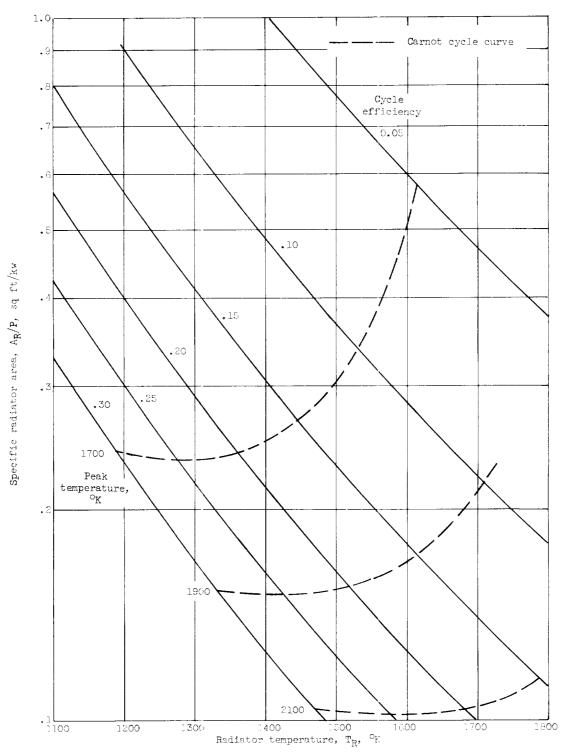


Figure 9. - Curves of specific radiator area as function of radiator temperature for constant values of efficiency and for Carnot cycles with fixed peak temperatures. Radiator emissivity, 0.9.

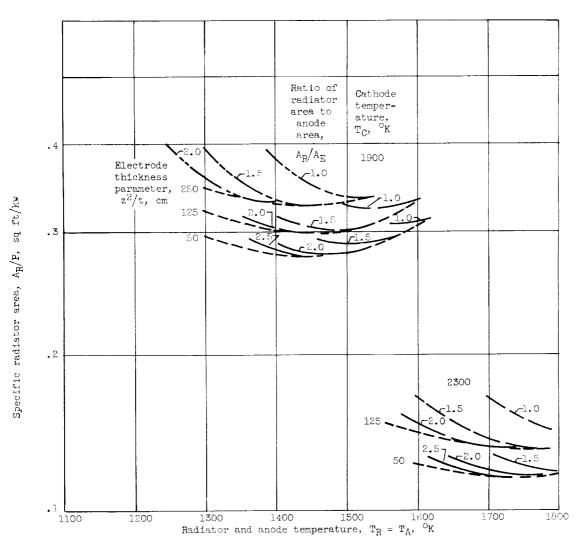


Figure 10. - Contours of fixed ratio of radiator area to anode area.

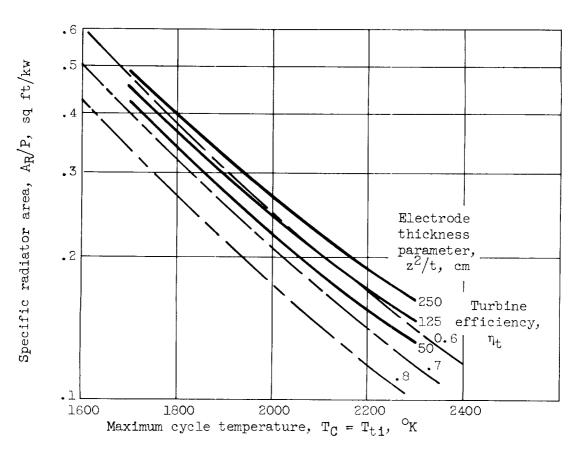
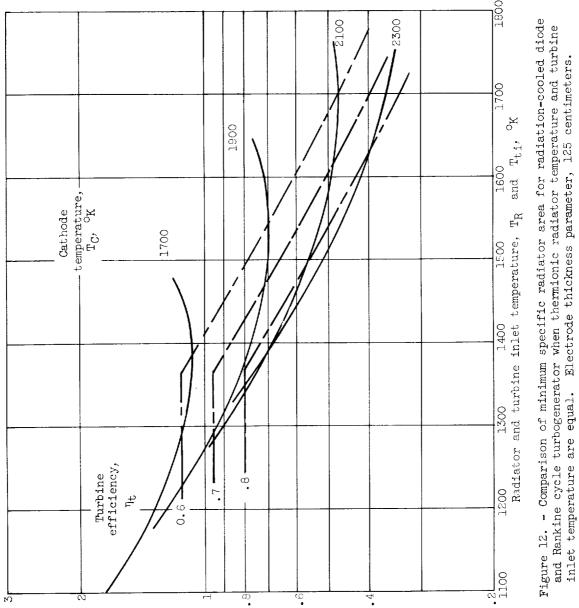


Figure 11. - Comparison of minimum specific radiator area for thermionic diode with that for Rankine cycle turbogenerator having turbine inlet temperature equal to cathode temperature. Thermionic radiator and anode temperatures are equal.



Specific radiator area, $A_{\rm R}/P_{\rm r}$ sq ft/kw

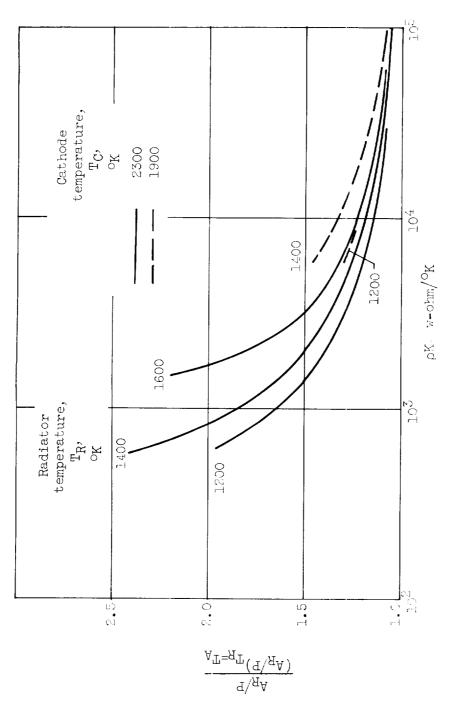


Figure 13. - Effect of insulator properties on minimum radiator area. Insulator thickness is optimized, electrode thickness parameter, 125 centimeters.

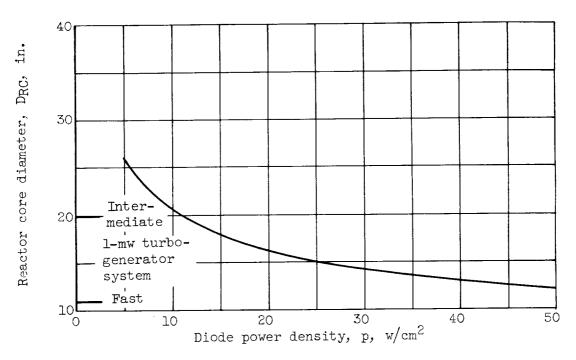
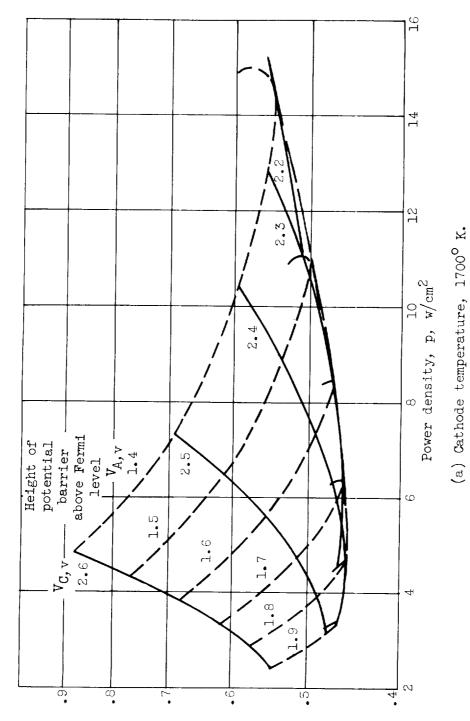


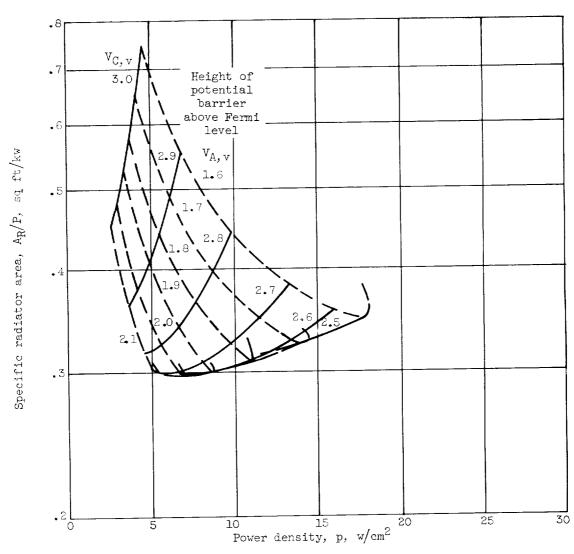
Figure 14. - Reactor core size required to house enough diodes with power density p to produce 1 megawatt of power.

Figure 15. - Specific radiator area and power density for equal radiator and anode

temperatures. Electrode thickness parameter, 125 centimeters.



Specific radiator area, A_{R}/P , sq ft/kw

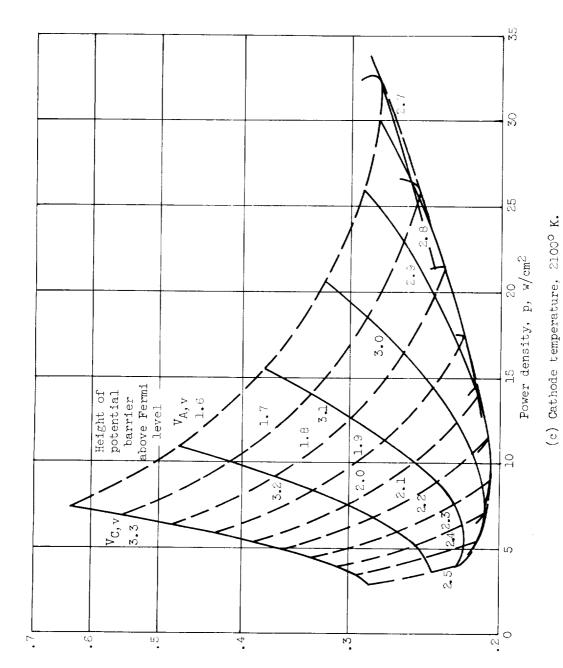


(b) Cathode temperature, 1900° K.

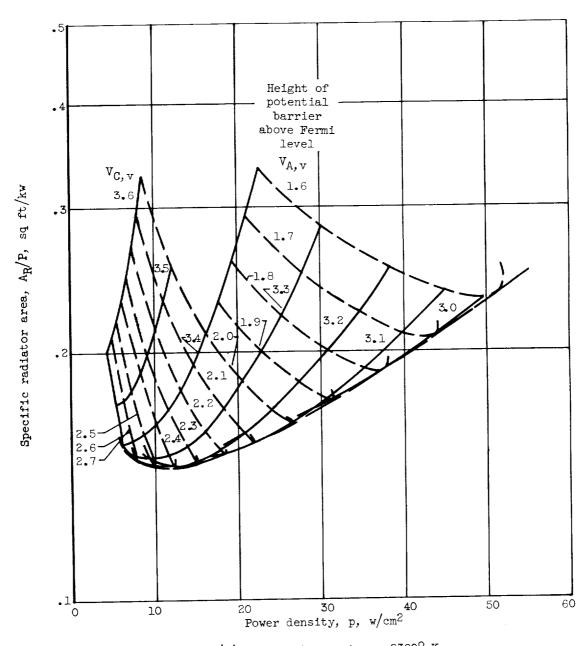
Figure 15. - Continued. Specific radiator area and power density for equal radiator and anode temperatures. Electrode thickness parameter, 125 centimeters.

Figure 15. - Continued. Specific radiator area and power density for equal radiator and anode temperatures. Electrode thickness parameter, 125 centimeters.



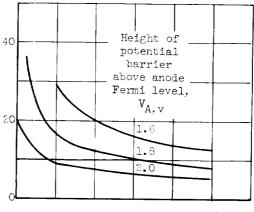


Specific radiator area, $A_{\overline{\rm R}}/P_{\nu}$ sq ft/kw

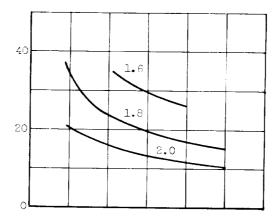


(d) Cathode temperature, 2300° K.

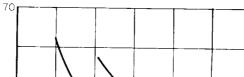
Figure 15. - Concluded. Specific radiator area and power density for equal radiator and anode temperatures. Electrode thickness parameter, 125 centimeters.



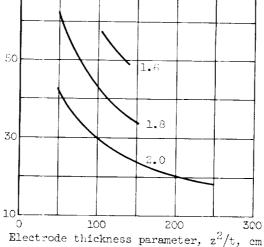
(a) Cathode temperature, 1900° K.



Power density, p, w/cm^2

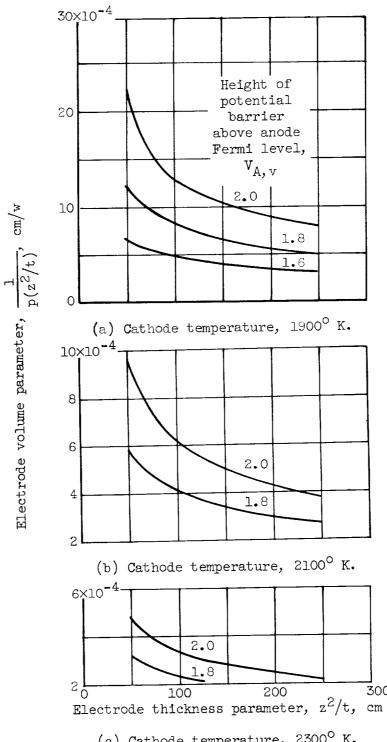


(b) Cathode temperature, 2100° K.



(c) Cathode temperature, 2300° K.

Figure 16. - Effect of electrode length and thickness on power density.



(c) Cathode temperature, 2300° K.

Figure 17. - Variation of electrode volume parameter for fixed length with electrode thickness parameter.

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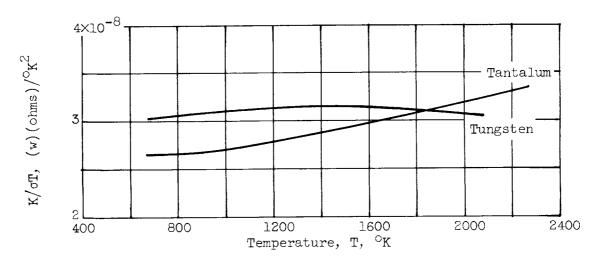


Figure 18. - Variation of $K/\sigma T$ with temperature for tungsten and tantalum. Data from reference 6.